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Consistent Comparisons of Real Incomes across Time and Space

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1. Introduction

Studies on catch-up and convergence of real incomes and comparative assessment of growth performance across countries require consistent estimates of real gross domestic product at constant prices. A major source of data on real GDP at constant prices is the Penn World Table (PWT) which regularly publishes constant price estimates of real GDP using fixed and chain-based methods. Similar estimates are also available from the work of Angus Maddison and the World Bank also produces such series for its World Development Indicators publication. Most of the currently produced series of real GDP at constant prices are derived by combining real GDP estimates at current prices from the International Comparison Program (ICP) benchmark data and observed growth rates in real GDP in respective countries. The existing approach combines the use of *reference prices at a point in time*, with *national/country prices over time*.

The main objective of this paper is to develop an analytical framework for making real income comparisons across time and space based on the concept of real income comparisons at constant prices. Suppose we have estimates of real incomes in two periods, and across multiple countries – similar to the current practice of compiling PPPs and real incomes for different benchmarks. Typically, benchmark estimates of real GDP are at current prices and are not comparable across the benchmarks. How can these measures of real incomes be compared?

The approach advocated here builds on the recent work of Neary (2004) which develops a framework to construct the Geary-Allen International Accounts (GAIA) system. In developing a framework based on economic approach, we *will not* start with fixed-weight calculations such as Geary-Khamis used in the construction of the Penn World Table. Instead, we will start with flexible-weight calculations, as in the GAIA system of Neary (2004). That is, we assume that the expenditure function of a representative consumer is known, and ask how we should then make consistent comparisons of real incomes across time and space. This sort of ideal circumstance will be compared to the actual calculations that are done in PWT and by other researchers, whereby real GDPs at a point in time are updated to other years using national accounts growth rates. Our aim is to show whether there are consistent biases in this approach, as compared to a consistent comparison that uses references price across countries *and* over time.

2. Notation and Basic Framework

We consider the case where there are M countries indexed by j = 1,2,…,M; along with N commodities labeled i = 1,2,…, N; and two time periods t-1 and t. Price and quantity data are represented by vectors $\mathbf{p}_{js}$ and $\mathbf{q}_{js}$ for countries $j=1,2,\ldots,M$ and $s = t-1$ and t. Total expenditure in domestic prices is denoted by $z_j = \mathbf{p}_j'\mathbf{q}_j$ **.** Following the economic approach, we make use of an expenditure function depending on a vector of prices, $\mathbf{p}$, and a specific utility level u:

$$e(\mathbf{p},u) = Min_{\mathbf{q}}\left\{\mathbf{p}'\mathbf{q} : U(\mathbf{q}) \geq u\right\}.$$

All the observed expenditures are *assumed* to be optimal for the utility levels observed. That is, $z_j = \mathbf{p}_j'\mathbf{q}_j = e(\mathbf{p}_j, U_j(\mathbf{q}_j))$ in both periods t-1 and t.

We will suppose that we have calculations from an expenditure function and reference prices $\boldsymbol{\pi}_t$ in each period:

$$e(\boldsymbol{\pi}_s, u_{js}) = \boldsymbol{\pi}_s'\mathbf{q}^*_{js}, \text{ for } j = 1,\ldots,M, \text{ and } s = t-1, t. \tag{1}$$

We do not assume that the reference prices are GAIA, necessarily. However, we do presume that (1) is known only up to some normalization, so that really we just know the ratios:

$$\frac{e(\boldsymbol{\pi}_s, u_{js})}{e(\boldsymbol{\pi}_s, u_{ks})} = \frac{\boldsymbol{\pi}_s'\mathbf{q}^*_{js}}{\boldsymbol{\pi}_s'\mathbf{q}^*_{ks}}, \tag{1'}$$

for some benchmark country k.

To consistently compare real incomes across countries *and* time, we need to have a "constant" reference price vector. We will consider:

$$\ln \pi_{i\alpha} = \alpha \ln \pi_{it} + (1-\alpha)\ln \pi_{it-1}, \text{ for } i = 1,\ldots,N. \tag{2}$$

That is, the common reference prices are a weighted geometric mean of the reference prices in each period: when $\alpha=0$ we are using period t-1 prices, and when $\alpha=1$ we are using period t prices. In most empirical applications, $\alpha = 0.5$ would be a natural choice. Use of $\boldsymbol{\pi}_\alpha$ provides for a more general class of constant-price comparisons.

Then if we have information on each period like in (1) or (1'), the question is how we can calculate:

$$\frac{e(\boldsymbol{\pi}_\alpha, u_{js})}{e(\boldsymbol{\pi}_\alpha, u_{ks})}, \text{ for } j = 1,\ldots,M, \text{ and } s = t\text{-}1,\ t. \tag{3}$$

Note that instead of computing the cross-country comparisons at a constant reference prices, we can instead compute the time-series comparisons of change in real incomes or utility levels over the period *t-1* to *t*.

$$\frac{e(\boldsymbol{\pi}_\alpha, u_{jt})}{e(\boldsymbol{\pi}_\alpha, u_{jt-1})}, \text{ for } j = 1,\ldots,M. \tag{4}$$

We call (3) and (4) together the "constant-price time-space comparison." The indexes used in (3) and (4) are essentially the *Allen quantity indices* and are preferred here as they are consistent with the concept of constant-price real income comparisons.

3. Real Income Comparisons with Translog Expenditure Function

In this paper we make use of the translog specification for the expenditure function. The expenditure function is specified as:

$$\ln e(\mathbf{p},u) = \beta_0 + \beta' \ln \mathbf{p} + \tfrac{1}{2} \ln \mathbf{p'} \Gamma \ln \mathbf{p} + \ln \mathbf{p'} \boldsymbol{\delta} \ln u + \eta \ln u + \phi (\ln u)^2 . \qquad (5)$$

This expenditure function is introduced by Diewert (1976, p. 122) and corresponds to a non-homothetic utility function if $\eta \neq 0$ or $\phi \neq 0$. Denoting the elements of Γ by γ_{ih}, with $\gamma_{ih} = \gamma_{hi}$, homogeneity of degree one in prices requires that $\sum_i \beta_i = 1$ and $\sum_i \delta_i = \sum_i \gamma_{ih} = 0$. For $\phi = 0$, the budget shares obtained are similar but not identical to the Almost Ideal Demand System (AIDS). Likewise, for $\phi \neq 0$ there is an additional quadratic term in utility appearing in (5), but that term differs from what is used in the Quadratic AIDS (QUAIDS); see section 4.

The following lemma provides a link between the constant-price comparisons using $\boldsymbol{\pi}_\alpha$ in equations (3) and (4) with the more familiar constant price comparisons that make use of period t or period $t\text{-}1$ price vectors, $\boldsymbol{\pi}_{t-1}$ and $\boldsymbol{\pi}_t$.

Lemma 1

If the expenditure function is translog, then for any pair of positive price vectors, $\boldsymbol{\pi}_{t-1}$ and $\boldsymbol{\pi}_t$, and $0 \leq \alpha \leq 1$,

$$\frac{e(\boldsymbol{\pi}_\alpha, u_{jt})}{e(\boldsymbol{\pi}_\alpha, u_{ks})} = \left[\frac{e(\boldsymbol{\pi}_t, u_{jt})}{e(\boldsymbol{\pi}_t, u_{ks})} \right]^{\alpha} \left[\frac{e(\boldsymbol{\pi}_{t-1}, u_{jt})}{e(\boldsymbol{\pi}_{t-1}, u_{ks})} \right]^{1-\alpha} \qquad (6)$$

Proof: Direct substitution after taking logs of (6), and using (2) and (4). QED

If we choose $k = j$ and $s = t\text{-}1$, then (6) states that the constant-price comparison of real incomes over time is a weighted geometric mean of the real income comparisons at reference prices π_{t-1} and π_t. The problem is that the denominator of the first term in (6) and the numerator of the second term are unobserved, i.e. depend on reference prices and utilities in different periods. As a first step towards overcoming this difficulty, let $P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha)$ denote the *j*-th country-specific reference-price index that is *defined* to satisfy:

$$\frac{e(\boldsymbol{\pi}_\alpha, u_{jt})}{e(\boldsymbol{\pi}_\alpha, u_{jt-1})} \equiv \left[\frac{e(\boldsymbol{\pi}_t, u_{jt})}{e(\boldsymbol{\pi}_{t-1}, u_{jt-1})}\right] / P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha) . \qquad (7)$$

That is, we are supposing that we can *deflate the observed change* in real incomes at current prices, which is the ratio $e(\boldsymbol{\pi}_t, u_{jt}) / e(\boldsymbol{\pi}_{t-1}, u_{jt-1})$, by the factor $P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha)$, to obtain the desired constant-price change in real income $e(\boldsymbol{\pi}_\alpha, u_{jt}) / e(\boldsymbol{\pi}_\alpha, u_{jt-1})$.

It should be stressed that $P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha)$ is a "theoretical" price index that depends on properties of the expenditure function, i.e. it is not necessarily measured with observable data. We can characterize this theoretical index with the following result:

Lemma 2

If the expenditure function is translog, then for any pair of positive price vectors, π_{t-1} and π_t, and $0 \le \alpha \le 1$, the deflator $P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha)$ defined by (7) equals,

$$P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha) = \left[\frac{e(\boldsymbol{\pi}_t, u_{jt-1})}{e(\boldsymbol{\pi}_{t-1}, u_{jt-1})}\right]^\alpha \left[\frac{e(\boldsymbol{\pi}_t, u_{jt})}{e(\boldsymbol{\pi}_{t-1}, u_{jt})}\right]^{1-\alpha} = \left[\frac{e(\boldsymbol{\pi}_t, u_{j\alpha})}{e(\boldsymbol{\pi}_{t-1}, u_{j\alpha})}\right], \qquad (8a)$$

where the reference utility $u_{j\alpha}$ is defined by:

$$\ln u_{j\alpha} \equiv \alpha \ln u_{jt-1} + (1-\alpha) \ln u_{jt} . \qquad (8b)$$

Proof: The first equality follows from Lemma 1 by using $j = k$, and $s = t\text{-}1$, along with (7). Taking logs of that result and cancelling common terms, the second equality follows. QED

This result is quite interesting in that the price index in (8) is a weighted geometric mean of the Laspeyres-Konus and Paasche-Konus price indexes associated with price vectors π_{t-1} and π_t respectively, and that weighted geometric mean equals, in turn, the ratio of expenditures at the reference prices π_{t-1} and π_t and the reference utility $u_{j\alpha}$. For $\alpha = ½$, the result in (8) combines the theorem in Diewert (1976, p. 122), which uses the reference utility $u_{j0.5}$, with the "translog identity" of Caves, Christensen and Diewert (1982, p. 1412). In the case of $\alpha = ½$, the theoretical index $P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha)$ equals the conventional Törnqvist price index. The next result shows how this results can be extended to allow for $\alpha \neq ½$:

<u>Lemma 3</u>

If the expenditure function is translog, then for any pair of positive price vectors, π_{t-1} and π_t, and $0 \leq \alpha \leq 1$, the theoretical price index $P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha)$ equals:

$$\ln P_j^*(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \alpha) = \ln P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}_{jt-1}^*, \mathbf{s}_{jt}^*) + \left(\ln \overline{\boldsymbol{\pi}} - \ln \boldsymbol{\pi}_\alpha\right)' \boldsymbol{\Gamma}\left(\ln \overline{\boldsymbol{\pi}} - \ln \boldsymbol{\pi}_\alpha\right), \qquad (9a)$$

where,

$$\ln P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}_{jt-1}^*, \mathbf{s}_{jt}^*) \equiv [\alpha \mathbf{s}_{jt-1}^* + (1-\alpha)\mathbf{s}_{jt}^*]'(\ln \boldsymbol{\pi}_t - \ln \boldsymbol{\pi}_{t-1}), \qquad (9b)$$

and $\boldsymbol{\pi}_\alpha$ is defined as in (2); $\ln \overline{\boldsymbol{\pi}} = 0.5\left(\ln \boldsymbol{\pi}_t + \ln \boldsymbol{\pi}_{t-1}\right)$; and $\mathbf{s}_{jt-1}^*$ are $\mathbf{s}_{jt}^*$ are the expenditure shares for each country at the reference prices,

$$\mathbf{s}_{j\tau}^* = \partial \ln e(\boldsymbol{\pi}_\tau, u_{j\tau}) / \partial \ln \boldsymbol{\pi}_\tau = \boldsymbol{\beta} + \boldsymbol{\Gamma} \ln \boldsymbol{\pi}_\tau + \boldsymbol{\delta} \ln u_{j\tau}, \quad \tau = t\text{-}1, t. \qquad (9c)$$

Proof: From the theorem in Diewert (1976, p. 122), the right-hand side of (8a) is the Törnqvist index given by:

$$\left[\frac{e(\boldsymbol{\pi}_t, u_{j\alpha})}{e(\boldsymbol{\pi}_{t-1}, u_{j\alpha})}\right] = P_{0.5}(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}_{\alpha jt-1}^*, \mathbf{s}_{\alpha jt}^*),$$

where $\mathbf{s}^*_{\alpha jt-1}$ and $\mathbf{s}^*_{\alpha jt}$ are equilibrium shares associated with $u_{j\alpha}$, given by:

$$\mathbf{s}^*_{\alpha j\tau} = \partial \ln e(\boldsymbol{\pi}_\tau, u_{j\alpha}) / \partial \ln \boldsymbol{\pi}_\tau = \boldsymbol{\beta} + \boldsymbol{\Gamma} \ln \boldsymbol{\pi}_\tau + \boldsymbol{\delta} \ln u_{j\alpha}, \quad \tau = t\text{-}1,\ 1. \tag{10}$$

Comparing these shares with shares $\mathbf{s}^*_{jt-1}$ and $\mathbf{s}^*_{jt}$ and re-arranging terms, we can derive the expression in (9). QED

Notice that for $\alpha = ½$, the index $P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}^*_{jt-1}, \mathbf{s}^*_{jt})$ is a conventional Törnqvist price index defined over the reference prices and optimal shares. In that case, Lemma 3 is similar to the "translog identity" of Caves, Christensen and Diewert (1982) or results in Diewert and Morrison (1986). For $\alpha \neq ½$ the index $P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}^*_{jt-1}, \mathbf{s}^*_{jt})$ is still measured with the reference prices and optimal shares, but no longer uses the arithmetic average of the shares: instead, for $\alpha=0$ the shares in period t are used, and $\alpha=1$ the shares in period $t\text{-}1$ are used. When $\alpha \neq ½$ there is an additional term that enters (9a), involving the matrix $\boldsymbol{\Gamma}$. That term is not measurable without knowledge of the parameters of the expenditures function. However, because this extra term does not depend on the country index j, we will find that it cancels out in our results below.

Lemma 3, combined with (7), shows that we can make some headway on measuring the constant-price time-series comparison in (4). But in fact, we can *also* make headway on the cross-country comparison in (3). Taking the ratio of (7) for countries j and k, we obtain what we shall call the *constant-reference-price* (CRP) approach to measuring the relative change in real income between countries j and k:

$$\text{CRP} \equiv \left[\frac{e(\boldsymbol{\pi}_\alpha, u_{jt})}{e(\boldsymbol{\pi}_\alpha, u_{kt})}\right] \Big/ \left[\frac{e(\boldsymbol{\pi}_\alpha, u_{jt-1})}{e(\boldsymbol{\pi}_\alpha, u_{kt-1})}\right] = \left[\frac{e(\boldsymbol{\pi}_\alpha, u_{jt})}{e(\boldsymbol{\pi}_\alpha, u_{jt-1})}\right] \Big/ \left[\frac{e(\boldsymbol{\pi}_\alpha, u_{kt})}{e(\boldsymbol{\pi}_\alpha, u_{kt-1})}\right]$$

$$= \left\{\left[\frac{e(\boldsymbol{\pi}_t, u_{jt})}{e(\boldsymbol{\pi}_t, u_{kt})}\right] \Big/ \left[\frac{e(\boldsymbol{\pi}_{t-1}, u_{jt-1})}{e(\boldsymbol{\pi}_{t-1}, u_{kt-1})}\right]\right\} \Big/ \frac{P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}^*_{jt-1}, \mathbf{s}^*_{jt})}{P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}^*_{kt-1}, \mathbf{s}^*_{kt})}, \tag{11}$$

where the final equality is obtained because the additional term involving the matrix Γ cancels out. From (11), we see that the comparison of real income at *constant* reference prices $\boldsymbol{\pi}_\alpha$, on the left, equals the comparison at "current" reference prices $\boldsymbol{\pi}_t$ and $\boldsymbol{\pi}_{t-1}$ on the right, *adjusted by* the ratio of price indexes P_α , reflecting how the change in reference prices is evaluated with country *j* average shares versus country *k* average shares. We have already assumed that the expenditures at references prices $\boldsymbol{\pi}_t$ and $\boldsymbol{\pi}_{t-1}$ are also observed, and the price indexes P_α for country *j* and *k* are measured as in (9b). So (11) shows how the change in real expenditures at constant-reference-prices $\boldsymbol{\pi}_\alpha$, or CRP, can be measured. This is summarized in the next result, which also shows how the CRP comparison is related to underlying utility levels:

Theorem 1

The change in constant-reference-price real income from period *t-1* to *t*, in country *j* relative to country *k*, is measured by (11). This expression is related to utility levels by:

$$\text{CRP} = \left(\frac{u_{jt}/u_{jt-1}}{u_{kt}/u_{kt-1}}\right)^{(\ln \boldsymbol{\pi}_\alpha^{'}\boldsymbol{\delta}+\eta)}\left[\frac{(u_{jt}/u_{jt-1})^{\phi\ln(u_{jt}u_{jt-1})}}{(u_{kt}/u_{kt-1})^{\phi\ln(u_{kt}u_{kt-1})}}\right]. \qquad (12)$$

Thus, if $\phi = 0$ and $(\ln \boldsymbol{\pi}_\alpha^{'}\boldsymbol{\delta}+\eta) > 0$, then the change in constant-reference-price real income exceeds unity if and only if $(u_{jt}/u_{jt-1}) > (u_{kt}/u_{kt-1})$, so that CRP is an accurate measure of the change in relative utilities. When $\phi \neq 0$ then the comparison of the change in constant-reference-price real income and the change in utility is offset by the final term appearing in (12).

Proof: Given the result in equation (8), then we can write (11) in logs as:

$$\ln\left\{\left[\frac{e(\boldsymbol{\pi}_{\alpha},u_{jt})}{e(\boldsymbol{\pi}_{\alpha},u_{kt})}\right]\Big/\left[\frac{e(\boldsymbol{\pi}_{\alpha},u_{jt-1})}{e(\boldsymbol{\pi}_{\alpha},u_{kt-1})}\right]\right\}$$

$$= \ln\left\{\left[\frac{e(\boldsymbol{\pi}_{t},u_{jt})}{e(\boldsymbol{\pi}_{t-1},u_{jt-1})}\right]\Big/\left[\frac{e(\boldsymbol{\pi}_{t},u_{j\alpha})}{e(\boldsymbol{\pi}_{t-1},u_{j\alpha})}\right]\right\} - \ln\left\{\left[\frac{e(\boldsymbol{\pi}_{t},u_{kt})}{e(\boldsymbol{\pi}_{t-1},u_{kt-1})}\right]\Big/\left[\frac{e(\boldsymbol{\pi}_{t},u_{k\alpha})}{e(\boldsymbol{\pi}_{t-1},u_{k\alpha})}\right]\right\}$$

$$= \ln\boldsymbol{\pi}_t'\boldsymbol{\delta}(\ln u_{jt} - \ln u_{j\alpha}) + \eta(\ln u_{jt} - \ln u_{j\alpha}) + \phi[(\ln u_{jt})^2 - (\ln u_{j\alpha})^2]$$

$$- \{\ln\boldsymbol{\pi}_{t-1}'\boldsymbol{\delta}(\ln u_{jt-1} - \ln u_{j\alpha}) + \eta(\ln u_{jt-1} - \ln u_{j\alpha}) + \phi[(\ln u_{jt-1})^2 - (\ln u_{j\alpha})^2]\}$$

$$- \{\ln\boldsymbol{\pi}_t'\boldsymbol{\delta}(\ln u_{kt} - \ln u_{k\alpha}) + \eta(\ln u_{kt} - \ln u_{k\alpha}) + \phi[(\ln u_{kt})^2 - (\ln u_{k\alpha})^2]\}$$

$$+ \ln\boldsymbol{\pi}_{t-1}'\boldsymbol{\delta}(\ln u_{kt-1} - \ln u_{k\alpha}) + \eta(\ln u_{kt-1} - \ln u_{k\alpha}) + \phi[(\ln u_{kt-1})^2 - (\ln u_{k\alpha})^2]$$

$$= (\ln\boldsymbol{\pi}_{\alpha}'\boldsymbol{\delta} + \eta)(\ln u_{jt} - \ln u_{jt-1}) + \phi(\ln u_{jt} - \ln u_{jt-1})(\ln u_{jt} + \ln u_{jt-1})$$

$$- [(\ln\boldsymbol{\pi}_{\alpha}'\boldsymbol{\delta} + \eta)(\ln u_{kt} - \ln u_{kt-1}) + \phi(\ln u_{kt} - \ln u_{kt-1})(\ln u_{kt} + \ln u_{kt-1})]$$

leading to the result in (12). QED

In the next sections we will implement the constant-reference-price formula in (11), which we can think of as a theoretically "ideal" comparisons of real incomes across countries and over time. Of course, such an ideal comparison is not what is implemented in practice. Instead, it is common for researchers (including PWT) to update the benchmark comparison of a single years using growth rates of real expenditure from national accounts data. We shall refer to this procedure as the *national-price-approach,* or NPA, to distinguish it from the *constant-reference-price* approach. Specifically, instead of using reference prices in (11) we use national prices $\mathbf{p}_{jt}$ and likewise for country k and period t-1, obtaining:

$$\text{NPA} \equiv \left\{\left[\frac{e(\mathbf{p}_{jt},u_{jt})}{e(\mathbf{p}_{kt},u_{kt})}\right]\Big/\left[\frac{e(\mathbf{p}_{t-1},u_{jt-1})}{e(\mathbf{p}_{kt-1},u_{kt-1})}\right]\right\}\Big/\frac{P_{0.5}(\mathbf{p}_{jt-1},\mathbf{p}_{jt},\mathbf{s}_{jt-1},\mathbf{s}_{jt})}{P_{0.5}(\mathbf{p}_{kt-1},\mathbf{p}_{kt},\mathbf{s}_{kt-1},\mathbf{s}_{kt})}, \tag{13}$$

where $\mathbf{s}_{j\tau}$ denotes the observed expenditure share in country j, $\tau = t\text{-}1, t$. The expenditures appearing in (13) are just the ratio of observed expenditure in countries *j* and *k*, for periods *t-1* and *t*. The index $P_{0.5}(\mathbf{p}_{jt-1}, \mathbf{p}_{jt}, \mathbf{s}_{jt-1}, \mathbf{s}_{jt})$ is the conventional Törnqvist price index. So (13) is simply the growth in real expenditure from national accounts data.

When comparing NPA and CRP, we make use of the fact that the utility levels appearing in (11) and (13) are identical. In other words, the representative consumer stays on the same indifference curve in each country, and only the prices vary: either national prices or reference prices. The next result extends Theorem 1 to the use of national prices, and also provides the comparison between the two approaches:

Theorem 2

The national-price comparison approach to computing real expenditure growth rates, or NPA, is measured by (13) and is related to utility levels by:

$$\text{NPA} = \left(\frac{u_{jt}/u_{jt-1}}{u_{kt}/u_{kt-1}}\right)^{\eta}\left[\frac{(u_{jt}/u_{jt-1})^{\ln\bar{\mathbf{p}}_j\boldsymbol{\delta}+\phi\ln(u_{jt}u_{jt-1})}}{(u_{kt}/u_{kt-1})^{\ln\bar{\mathbf{p}}_k\boldsymbol{\delta}+\phi\ln(u_{kt}u_{kt-1})}}\right], \tag{14}$$

where $\ln\bar{\mathbf{p}}_j = 0.5\left(\ln\mathbf{p}_{jt} + \ln\mathbf{p}_{jt-1}\right)$. Thus, for α = ½, the comparison of NPA and CRP is:

$$\text{NPA/CRP} = \left(\frac{u_{jt}}{u_{jt-1}}\right)^{-(\ln\bar{\boldsymbol{\pi}}-\ln\bar{\mathbf{p}}_j)'\boldsymbol{\delta}}\left(\frac{u_{kt}}{u_{kt-1}}\right)^{(\ln\bar{\boldsymbol{\pi}}-\ln\bar{\mathbf{p}}_k)'\boldsymbol{\delta}}. \tag{15}$$

Proof:

From Diewert (1976, p. 122), we know that the Törnqvist index in (13) equals:

$$P_{0.5}(\mathbf{p}_{jt-1}, \mathbf{p}_{jt}, \mathbf{s}_{jt-1}, \mathbf{s}_{jt}) = \left[\frac{e(\mathbf{p}_t, \bar{u}_j)}{e(\mathbf{p}_{t-1}, \bar{u}_j)}\right],$$

where $\ln \bar{u}_j = 0.5(\ln u_{jt-1} + \ln u_{jt})$. Using this result in (13), we obtain:

$$\left\{\left[\frac{e(\mathbf{p}_{jt}, u_{jt})}{e(\mathbf{p}_{kt}, u_{kt})}\right] \Big/ \left[\frac{e(\mathbf{p}_{t-1}, u_{jt-1})}{e(\mathbf{p}_{kt-1}, u_{kt-1})}\right]\right\} \Big/ \frac{P_{0.5}(\mathbf{p}_{jt-1}, \mathbf{p}_{jt}, \mathbf{s}_{jt-1}, \mathbf{s}_{jt})}{P_{0.5}(\mathbf{p}_{kt-1}, \mathbf{p}_{kt}, \mathbf{s}_{kt-1}, \mathbf{s}_{kt})}$$

$$= \ln\left\{\left[\frac{e(\mathbf{p}_{jt}, u_{jt})}{e(\mathbf{p}_{jt-1}, u_{jt-1})}\right] \Big/ \left[\frac{e(\mathbf{p}_{jt}, \bar{u}_j)}{e(\mathbf{p}_{jt-1}, \bar{u}_j)}\right]\right\} - \ln\left\{\left[\frac{e(\mathbf{p}_{kt}, u_{kt})}{e(\mathbf{p}_{kt-1}, u_{kt-1})}\right] \Big/ \left[\frac{e(\mathbf{p}_{kt}, \bar{u}_k)}{e(\mathbf{p}_{kt-1}, \bar{u}_k)}\right]\right\}$$

$$= \ln \mathbf{p}'_{jt}\boldsymbol{\delta}(\ln u_{jt} - \ln \bar{u}_j) + \eta(\ln u_{jt} - \ln \bar{u}_j) + \phi[(\ln u_{jt})^2 - (\ln \bar{u}_j)^2]$$

$$- \{\ln \mathbf{p}'_{jt-1}\boldsymbol{\delta}(\ln u_{jt-1} - \ln \bar{u}_j) + \eta(\ln u_{jt-1} - \ln \bar{u}_j) + \phi[(\ln u_{jt-1})^2 - (\ln \bar{u}_j)^2]\}$$

$$- \{\ln \mathbf{p}'_{kt}\boldsymbol{\delta}(\ln u_{kt} - \ln \bar{u}_k) + \eta(\ln u_{kt} - \ln \bar{u}_k) + \phi[(\ln u_{kt})^2 - (\ln \bar{u}_k)^2]\}$$

$$+ \ln \mathbf{p}'_{kt-1}\boldsymbol{\delta}(\ln u_{kt-1} - \ln \bar{u}_k) + \eta(\ln u_{kt-1} - \ln \bar{u}_k) + \phi[(\ln u_{kt-1})^2 - (\ln \bar{u}_k)^2]$$

$$= (\ln \bar{\mathbf{p}}'_j\boldsymbol{\delta} + \eta)(\ln u_{jt} - \ln u_{jt-1}) + \phi(\ln u_{jt} - \ln u_{jt-1})(\ln u_{jt} + \ln u_{jt-1})$$

$$- [(\ln \bar{\mathbf{p}}'_k\boldsymbol{\delta} + \eta)(\ln u_{kt} - \ln u_{kt-1}) + \phi(\ln u_{kt} - \ln u_{kt-1})(\ln u_{kt} + \ln u_{kt-1})]$$

which gives us the result in (14). Dividing (14) and (12) for α = ½, we obtain (15). QED

Theorem 2 allows us to interpret and examine the deviations of the constant-reference-price approach proposed here and the usual national-price approach to the comparison of real incomes over time. To interpret (15) it is useful to consider some special cases.

Remark (a): Suppose that *the utilities of both countries are rising, and that the reference prices* $\ln \bar{\pi}$ *are an arithmetic average of the average national prices* $\ln \bar{\mathbf{p}}_j$ and $\ln \bar{\mathbf{p}}_k$ *of countries j and k*. Then the two exponents in (15) are equal. If the prices of country k, say the U.S., are highest for luxury goods ($\delta_i > 0$), then the exponents in (15) are *negative,* so NPA/CRP < 1. In that case, NPA will *understate the growth* in real income of countries poorer than the U.S. We know from Neary's (2004) work that Geary-Khamis *understates* the disparity in real incomes as compared to the GAIA or expenditure-function approach. So in PWT, for example, which uses Geary-Khamis and also the national-price approach in (13), we have *both* an understatement of the disparity in real incomes, and an *understatement* of the growth in real incomes; in this sense, the two sources of error are “consistent.”

Remark (b): Suppose that *the utility of the poorer country j is falling, and the reference prices* $\ln\bar{\pi}$ *are equal to those of the rich country k.* This case applies to some low-income countries, such as in Africa, when faced with reference prices close to the U.S. In this case only the first term on the right of (15) applies, and if the prices of country k are highest for luxury goods ($\delta_i > 0$), then the exponent on the first term is again *negative*. Since we assume that utility is falling for country j, it follows that NPA/CRP >1. So in this case we find an *overstatement* in the real income growth of country j.

Putting together Remarks (a) and (b), we see that the national accounts calculation of real income growth is *biased towards unity, or zero growth.* This result is confirmed by the empirical results reported in the next section.

4. Empirical Application

In the empirical application of the approach described above, we consider the 1980 and 1996 benchmarks as the two periods (*t-1* and *t*). The data for 1980 are taken from Neary (2004), who adopts the data from Phase IV of the United Nations International Comparison Project (ICP) (United Nations, 1986). There are 60 countries and 11 categories of products: food, beverages, tobacco, clothing and footwear, gross rents and water charges, fuel and power, household furnishings, medical and health services, transport and communications, recreation and education, and miscellaneous goods and services.

The data for 1996 are that used to obtain the 1996 benchmark of PWT, and include 99 countries and 29 product categories. To make consistent comparison over time, we aggregate those 29 categories into the same 11 broader categories as in the 1980 data used by Neary (2004). We end up with 48 countries appearing in both 1980 and 1996 datasets, so our time-space comparison of income will be focused on those 48 countries.

We follow Neary's econometric strategy in estimating a Quadratic Almost Ideal Demand (QUAIDS) system specified as:

$$\ln e(\mathbf{p},u) = \ln g(\mathbf{p}) + [d(\mathbf{p})\ln u]/[1-\lambda(\mathbf{p})\ln u], \tag{16a}$$

where, $$\ln g(\mathbf{p}) = \beta_0 + \beta' \ln \mathbf{p} + \tfrac{1}{2}\ln \mathbf{p'}\Gamma \ln \mathbf{p}, \tag{16b}$$

and, $$\ln d(\mathbf{p}) = \ln \eta + \sum_i \delta_i \ln p_i,\ \lambda(\mathbf{p}) = \sum_i \lambda_i \ln p_i, \text{ with } \sum_i \lambda_i = 0. \tag{16c}$$

Notice that for $\delta_i = \lambda_i = 0$, we obtain a simple translog function from (16), also referred to as the Homothetic AIDS (HAIDS), which is identical to (5) when $\delta_i = \phi = 0$. For the non-homothetic cases, with $\delta_i \neq 0$ or $\lambda_i \neq 0$, the expenditure function in (16) is not identical to (5).

Denoting the expenditure at domestic prices of country j by $z_j = \mathbf{p}_j'\mathbf{q}_j$, the expenditure shares on each product are obtained as:

$$s_{ij} = \beta_i + \sum_h \gamma_{ih} \ln p_{hj} + \delta_i \ln[z_j / g(\mathbf{p}_j)] + [\lambda_i / d(\mathbf{p}_j)]\{\ln[z_j / g(\mathbf{p}_j)]\}^2,$$

The indirect utility function taking the form (Banks et al, 1997):

$$\ln V_j = \left\{ \left[\frac{\ln z_j - \ln g(\mathbf{p}_j)}{d(\mathbf{p}_j)} \right]^{-1} + \lambda(\mathbf{p}_j) \right\}^{-1}.$$

For the AIDS system this is simplified as $\ln V_j = [\ln z_j - \ln g(\mathbf{p}_j)]/d(\mathbf{p}_j)$. So given estimated parameters and nominal expenditure z_j, we can calculate the utility levels.

We follow Neary (2004) to adopt the semi-flexible approach to estimate the demand systems as described by the above budget function[1]. The sample include 11 product categories, and 60 or 99 countries, respectively for 1980 and 1996.The estimated parameters are then used in a maximum likelihood estimation to get the estimated reference prices, respectively for 1980 and 1999. Those reference prices $\boldsymbol{\pi}_t$ and $\boldsymbol{\pi}_{t-1}$ are reported in Table 1.

The upper panel of Table 1 is directly from Neary (2004, Table II), while the lower panel gives the corresponding 1996 references prices estimated respectively under the Geary method, HAIDS, AIDS, and QUAIDS. Because of the poor correlation between Geary prices and the translog expenditure function (the HAIDS), we won't report any further results under HAIDS in what follows.

Equation (11) and Theorem 1 suggests a channel of comparing real income across countries at constant reference prices (CRP). We could use the GAIA reference prices estimated using the almost ideal demand systems, as in Neary (2004). Choosing the United States as the comparison country, The first two columns of Table 2 give the expenditure using 1980 reference prices, under AIDS and QUAIDS assumptions, that is, $e(\boldsymbol{\pi}_{t-1}, u_{jt-1}) / e(\boldsymbol{\pi}_{t-1}, u_{us,t-1})$. This is exactly what is reported in Neary (2004, Table 1), except that we re-scale his expenditure by using the U.S. as the comparison country. The following two columns, (3) and (4), then give the analogous expenditure using the estimated 1996 reference prices, $e(\boldsymbol{\pi}_{t-1}, u_{jt}) / e(\boldsymbol{\pi}_{t-1}, u_{us,t})$. Columns (5) and (6) then provides the price deflator used in (7) and measured by (9), where we take $\alpha = ½$ such that $P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, s^*_{jt-1}, s^*_{jt})$ is actually the conventional Tornqvist price index. The last two columns calculate the CRP growth in expenditure, i.e. equation (11).

[1] See Neary (2004, Appendix D), available at: http://www.economics.ox.ac.uk/members/peter.neary/gaia/gaia.htm.

Notice that the price deflator using in (11), measured in logs, is:

$$\ln P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, s^*_{jt-1}, s^*_{jt}) - \ln P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, s^*_{kt-1}, s^*_{kt})$$

$$= \ln\left[\frac{e(\boldsymbol{\pi}_t, u_{j\alpha})}{e(\boldsymbol{\pi}_{t-1}, u_{j\alpha})}\right] - \ln\left[\frac{e(\boldsymbol{\pi}_t, u_{k\alpha})}{e(\boldsymbol{\pi}_{t-1}, u_{k\alpha})}\right]$$

$$= (\ln \boldsymbol{\pi}_t - \ln \boldsymbol{\pi}_{t-1})' \boldsymbol{\delta}(\ln u_{j\alpha} - \ln u_{k\alpha}) .$$

So choosing country k as the higher utility country, $u_{k\alpha} > u_{j\alpha}$, then,

$$\frac{P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}^*_{jt-1}, \mathbf{s}^*_{jt})}{P_\alpha(\boldsymbol{\pi}_{t-1}, \boldsymbol{\pi}_t, \mathbf{s}^*_{kt-1}, \mathbf{s}^*_{kt})} = \left(\frac{u_{j\alpha}}{u_{k\alpha}}\right)^{(\ln \boldsymbol{\pi}_t - \ln \boldsymbol{\pi}_{t-1})'\delta} >(<)\ 1 \text{ as } (\ln \boldsymbol{\pi}_t - \ln \boldsymbol{\pi}_{t-1})'\delta <(>)\ 0.$$

In Table 2, columns (5) and (6), this price deflator *exceeds* unity, meaning that the constant-price rise in real income in country j, relative to the U.S., is *less than* the current-price comparison, and is monotonically related to the spread between country j and k. This result is confirmed by the empirical results based on the 1980 and 1996 benchmark comparisons reported in the next columns of Table 2 where *all of the ratios are greater than unity and increase as we move towards poorer countries.*

Table 3 then shows the national price comparisons using real expenditures in 1980 and 1996, and the national price approach (NPA). The formula is given in equation (13). Column (1) gives the nominal per capita expenditure in 1996, denominated in U.S. dollars. Column (3) normalizes the 1996 expenditure in column (1), but takes the U.S. as the reference country. Column (2) gives the relative dollar expenditure of each country in 1980, relative to the U.S. Column (4) calculates the national price deflator using the conventional Tornqvist index, relative to the U.S. The last column gives the national-price real growth in expenditure, calculated by dividing column (3) by column (2), then divided by column (4).

Finally, Table 4 reports the comparison of growth in real income at constant reference prices versus that at national real prices. Column (1) and (2) are directly from the last two columns of Table 2, and column (3) is directly from the last column of Table 3. We then take ratio of the growth of real income at national prices relative to the growth of income at constant reference prices (respectively under AIDS and QUAIDS), resulting in the last two columns of Table 4.

The last two columns of Table 4 provide the comparison of national-price approach (NPA) to the use of constant-reference-prices (CRP). As summarized in Remark (a) after Theorem 2, when the utilities of both countries are rising and the reference prices are in-between the national prices, we should expect to see NPA/CRP < 1, so the national-price approach *understates* the growth in real income. That prediction is confirmed for all countries in Table 4 down to Bolivia (ranked 37th by nominal income in 1996). Next comes Nigeria, and suddenly we see that NPA/CRP > 1. That circumstance can be understood by Remark (b) after Theorem 2, under which the reference prices are close to those of the U.S. and the utility of the country in question is falling. In that case, we can find NPA/CRP > 1, as shown for a number of other African countries ranked lower than Nigeria.

Figure 1 summarizes our findings by graphing the national-price approach (NPA) to the growth rate of real income on the horizontal axis, and then the constant-reference-price (CRP) growth rate of real income on the vertical axis, separately for the AIDS and QUAIDS systems. It can be seen that the CRP growth of real income *exceed* the NPA growth for wealthier countries, but are *less than* the NPA growth for poorer countries. These results therefore demonstrate a systematic bias in the NPA method of computing real income growth over time.

5. Concluding remarks

In this paper we have provided an economic theoretic framework for making constant price real income comparisons over time and space. The approach is designed to extend the current work on cross-country comparisons of real incomes to consider comparisons over time. The approach proposed here provides a transitively consistent set of real income comparisons over space and time. The approach has the advantage that it can be easily linked to the current PWT and other approaches which make use of national growth rates to update benchmark comparisons to derive constant price real income comparisons. Further, the approach provides a consistent link between current and constant price real income comparisons and the underlying price index numbers. The approach is empirically implemented using data from the 1980 and 1996 benchmark data. Further work to extend the results presented in the paper is currently in progress.

References

Caves, Douglas W., Laurits R. Christensen and W. Erwin Diewert, 1982a, "The Economic Theory of Index Numbers and the Measurement of Input, Output and Productivity," *Econometrica*, 50(11), November, 1393-1414

Caves, Douglas W., Laurits R. Christensen and W. Erwin Diewert, 1982b, "Multilateral Comparisons of Output, Input, and Productivity Using Superlative Index Numbers," *Economic Journal*, 92, March, 73-86.

Diewert, W. Erwin, 1976, "Exact and Superlative Index Numbers," *Journal of Econometrics*, 4, 115-145.

Diewert, W. Erwin and Catherine J. Morrison, 1986, "Adjusting Outputs and Productivity Indexes for Changes in the Terms of Trade," *Economic Journal*, 96, 659-679.

Feenstra, Robert C., Alan Heston, Marcel P. Timmer, and Haiyan Deng, 2004, “Estimating Real Production and Expenditures Across Countries: A Proposal for Improving the Penn World Tables,” NBER Working Paper no. 10866.

Neary, Peter, 2004, "Rationalizing the Penn World Table: True multilateral indices for international comparisons of real income", *American Economic Review*, 94(5), December, 1411-1428,

Table 1: REFERENCE PRICES IMPLIED BY THE GEARY AND GAIA ESTIMATES

		Geary	HAIDS	AIDS	QUAIDS
	1980 π_{t-1}				
1	Food	1.068	0.309	0.898	0.884
2	Beverages	0.732	0.144	0.667	0.648
3	Tobacco	0.894	0.406	1.003	0.966
4	Clothing & Footware	1.010	0.471	1.011	0.959
5	Gross Rent and Water Charges	0.870	0.436	0.876	0.797
6	Fuel and Power	0.968	4.596	1.092	0.981
7	Household Furnishings	1.109	0.684	1.147	1.103
8	Medical and health Services	1.012	0.456	1.148	1.090
9	Transport and Communications	0.940	1.241	1.086	1.055
10	Recreation and Education	1.209	0.440	1.154	1.189
11	Miscellaneous	1.000	1.000	1.000	1.000
	Correlation with Geary Prices[a]		0.007	0.753	0.828
	1996 π_t				
1	Food	1.043	0.386	0.826	0.856
2	Beverages	0.907	1.525	0.728	0.817
3	Tobacco	0.960	1.663	1.101	1.063
4	Clothing & Footware	0.977	0.700	0.927	0.879
5	Gross Rent and Water Charges	0.698	1.911	0.780	0.895
6	Fuel and Power	0.912	0.268	0.938	0.992
7	Household Furnishings	0.915	1.561	1.046	0.917
8	Medical and health Services	1.046	0.969	1.003	1.057
9	Transport and Communications	0.942	1.249	0.948	0.941
10	Recreation and Education	0.958	2.280	0.950	0.993
11	Miscellaneous	1.000	1.000	1.000	1.000
	Correlation with Geary Prices[a]		-0.465	0.421	0.295

a. Simple correlation coefficient between Geary reference prices and GAIA references prices.

TABLE 2: REFERENCE PRICE COMPARISONS

	Country	Expenditure at Reference Prices 1980[a]		Expenditure at Reference Prices 1996[b]		Reference Price Deflator[c]		Growth in Expenditure[d]	
		AIDS	QUAIDS	AIDS	QUAIDS	AIDS	QUAIDS	AIDS	QUAIDS
1	Luxembourg	0.864	0.868	1.053	1.055	1.000	1.001	1.218	1.214
2	Japan	0.659	0.661	0.928	0.919	1.001	1.003	1.405	1.386
3	Norway	0.676	0.676	0.842	0.840	1.001	1.003	1.245	1.240
4	Denmark	0.840	0.838	0.820	0.831	1.001	1.001	0.975	0.990
5	Austria	0.781	0.777	0.827	0.837	1.001	1.002	1.059	1.075
6	Germany	0.858	0.852	0.764	0.768	1.001	1.001	0.890	0.901
7	**United States[e]**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
8	France	0.842	0.836	0.741	0.744	1.001	1.002	0.879	0.889
9	Belgium	0.842	0.834	0.766	0.771	1.001	1.002	0.909	0.922
10	Hong Kong	0.647	0.646	0.846	0.846	1.001	1.003	1.305	1.305
11	Finland	0.666	0.667	0.652	0.657	1.001	1.003	0.977	0.982
12	Netherlands	0.801	0.790	0.719	0.726	1.001	1.002	0.897	0.917
13	Israel	0.534	0.531	0.782	0.779	1.002	1.004	1.461	1.460
14	Italy	0.727	0.724	0.740	0.748	1.001	1.002	1.016	1.031
15	United Kingdom	0.740	0.734	0.741	0.750	1.001	1.002	0.999	1.019
16	Canada	1.018	1.030	0.830	0.834	1.000	1.000	0.815	0.810
17	Ireland	0.512	0.510	0.568	0.576	1.002	1.005	1.107	1.123
18	Spain	0.602	0.598	0.562	0.568	1.002	1.004	0.931	0.945
19	Greece	0.479	0.476	0.526	0.531	1.003	1.005	1.096	1.111
20	Portugal	0.392	0.389	0.559	0.566	1.003	1.006	1.419	1.447
21	Korea	0.173	0.165	0.548	0.547	1.006	1.011	3.144	3.273
22	Argentina	0.330	0.327	0.419	0.425	1.004	1.008	1.266	1.292
23	Uruguay	0.414	0.410	0.326	0.331	1.003	1.007	0.785	0.801
24	Chile	0.304	0.300	0.305	0.312	1.004	1.008	1.000	1.031
25	Brazil	0.329	0.327	0.261	0.268	1.004	1.008	0.788	0.812
26	Hungary	0.390	0.385	0.334	0.333	1.003	1.007	0.852	0.858
27	Dominica	0.206	0.200	0.183	0.179	1.006	1.011	0.881	0.886
28	Poland	0.352	0.346	0.242	0.237	1.004	1.008	0.684	0.679
29	Panama	0.222	0.217	0.202	0.205	1.005	1.010	0.904	0.932
30	Venezuela	0.409	0.402	0.189	0.195	1.003	1.007	0.460	0.482
31	Botswana	0.095	0.094	0.198	0.202	1.008	1.014	2.073	2.126
32	Peru	0.208	0.201	0.165	0.168	1.006	1.011	0.789	0.828
33	Tunisia	0.170	0.167	0.180	0.188	1.006	1.011	1.055	1.111
34	Ecuador	0.193	0.188	0.114	0.118	1.006	1.011	0.590	0.622
35	Philippines	0.153	0.145	0.129	0.129	1.007	1.012	0.837	0.883
36	Indonesia	0.078	0.072	0.118	0.119	1.009	1.015	1.495	1.636
37	Bolivia	0.124	0.119	0.081	0.081	1.007	1.013	0.651	0.668
38	Nigeria	0.056	0.053	0.013	0.013	1.010	1.014	0.231	0.244
39	Zimbabwe	0.073	0.072	0.086	0.091	1.009	1.015	1.173	1.252
40	Cote d`Ivoire	0.089	0.087	0.041	0.042	1.009	1.014	0.459	0.479
41	Cameroon	0.063	0.061	0.041	0.043	1.010	1.015	0.643	0.689
42	Senegal	0.062	0.062	0.038	0.039	1.010	1.015	0.606	0.620
43	Zambia	0.039	0.040	0.023	0.023	1.011	1.016	0.572	0.573

TABLE 2 (CONT.): REFERENCE PRICE COMPARISONS

	Country	Expenditure at Reference Prices 1980		Expenditure at Reference Prices 1996		Reference Price Deflator		Growth in Expenditure	
		AIDS	QUAIDS	AIDS	QUAIDS	AIDS	QUAIDS	AIDS	QUAIDS
44	Kenya	0.047	0.046	0.029	0.029	1.011	1.016	0.595	0.607
45	Mali	0.026	0.025	0.023	0.024	1.013	1.017	0.873	0.914
46	Madagascar	0.045	0.043	0.023	0.024	1.011	1.016	0.515	0.544
47	Malawi	0.034	0.035	0.020	0.021	1.012	1.016	0.590	0.579
48	Tanzania	0.020	0.019	0.013	0.013	1.014	1.017	0.667	0.673

a. the expenditure calculated using 1980 reference prices (i.e. Neary's estimates);
b. the expenditure calculated using 1996 reference prices;
c. the second term on the RHS of equation (11) --- the Tornqvist index for each country, relative to the U.S.;
d. Growth in expenditure using constant reference prices = (b/a)/c;
e. All expenditures are calculated relative to the U.S.

Table 3: NATIONAL PRICE COMPARISONS

	Country	Nominal per capita Expenditure in 1996 [a]	Relative Expenditure in 1980 [b]	Relative Expenditure in 1996 [b]	National Price Deflator [c]	National Real Growth in Expenditure [d]
1	Luxembourg	37236.397	0.994	1.359	1.183	1.155
2	Japan	36182.801	0.698	1.320	1.488	1.270
3	Norway	31694.567	0.904	1.157	1.140	1.122
4	Denmark	31531.256	1.122	1.151	1.109	0.925
5	Austria	28761.341	0.890	1.050	1.163	1.014
6	Germany	27869.892	1.128	1.017	1.044	0.864
7	**United States** [e]	**27401.837**	**1.000**	**1.000**	**1.000**	**1.000**
8	France	25587.309	1.043	0.934	1.080	0.829
9	Belgium	24592.522	1.047	0.897	0.997	0.859
10	Hong Kong	24145.454	0.436	0.881	1.668	1.212
11	Finland	22393.307	0.749	0.817	1.248	0.874
12	Netherlands	22155.645	0.977	0.809	0.953	0.868
13	Israel	20227.433	0.438	0.738	1.250	1.348
14	Italy	20165.848	0.616	0.736	1.257	0.950
15	United Kingdom	19892.412	0.804	0.726	0.934	0.966
16	Canada	19196.439	0.886	0.701	0.965	0.819
17	Ireland	16434.509	0.481	0.600	1.141	1.093
18	Spain	14917.894	0.526	0.544	1.184	0.874
19	Greece	12778.031	0.385	0.466	1.180	1.026
20	Portugal	12019.674	0.257	0.439	1.343	1.273
21	Korea	11796.324	0.135	0.430	1.344	2.381
22	Argentina	8310.075	0.454	0.303	0.611	1.092
23	Uruguay	5846.203	0.344	0.213	0.918	0.675
24	Chile	4863.060	0.235	0.177	0.800	0.943
25	Brazil	4765.295	0.193	0.174	1.333	0.674
26	Hungary	4296.772	0.149	0.157	1.438	0.733
27	Dominica	3666.512	0.124	0.134	1.245	0.868
28	Poland	3529.437	0.195	0.129	1.114	0.592
29	Panama	3035.387	0.149	0.111	0.898	0.831
30	Venezuela	2674.135	0.297	0.098	0.774	0.425
31	Botswana	2605.228	0.080	0.095	0.666	1.787
32	Peru	2539.953	0.099	0.093	1.370	0.682
33	Tunisia	2154.606	0.115	0.079	0.660	1.034
34	Ecuador	1663.824	0.120	0.061	1.071	0.474
35	Philippines	1221.336	0.066	0.045	0.878	0.768
36	Indonesia	1081.335	0.039	0.039	0.699	1.441
37	Bolivia	950.553	0.088	0.035	0.611	0.646
38	Nigeria	924.273	0.081	0.034	1.389	0.300
39	Zimbabwe	738.976	0.060	0.027	0.291	1.535
40	Cote d`Ivoire	692.903	0.101	0.025	0.453	0.552
41	Cameroon	639.275	0.076	0.023	0.426	0.724
42	Senegal	563.263	0.056	0.021	0.595	0.621
43	Zambia	401.884	0.048	0.015	0.501	0.610

Table 3 (CONT.): NATIONAL PRICE COMPARISONS

	Country	Nominal per capita Expenditure in 1996	Relative Expenditure in 1980	Relative Expenditure in 1996	National Price Deflator	National Real Growth in Expenditure
44	Kenya	347.725	0.037	0.013	0.521	0.657
45	Mali	324.483	0.022	0.012	0.606	0.908
46	Madagascar	306.059	0.038	0.011	0.714	0.414
47	Malawi	250.631	0.019	0.009	0.762	0.648
48	Tanzania	210.595	0.027	0.008	0.565	0.502

a. The total expenditure over all product categories, in US dollar ;
b. The total expenditure of each country in US dollar, relative to the U.S. for 1980 and 1996;
c. The second term on the RHS of equation (11) --- the Tornqvist index for each country, relative to the U.S.;
d. Growth in expenditure using national real prices;
e. All expenditures are calculated relative to the U.S.

Table 4: GROWTH IN REAL INCOME AT CONSTANT REFERENCE PRICES VERSUS AT NATIONAL PRICES

	Country	Growth in Expenditure [a]		National Real Growth in Expenditure [b]	Ratio	
		AIDS	QUAIDS		AIDS	QUAIDS
1	Luxembourg	1.218	1.214	1.155	0.948	0.951
2	Japan	1.405	1.386	1.270	0.904	0.916
3	Norway	1.245	1.240	1.122	0.901	0.905
4	Denmark	0.975	0.990	0.925	0.949	0.934
5	Austria	1.059	1.075	1.014	0.958	0.943
6	Germany	0.890	0.901	0.864	0.971	0.959
7	**United States**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
8	France	0.879	0.889	0.829	0.943	0.933
9	Belgium	0.909	0.922	0.859	0.945	0.932
10	Hong Kong	1.305	1.305	1.212	0.929	0.929
11	Finland	0.977	0.982	0.874	0.895	0.890
12	Netherlands	0.897	0.917	0.868	0.968	0.947
13	Israel	1.461	1.460	1.348	0.923	0.923
14	Italy	1.016	1.031	0.950	0.935	0.921
15	United Kingdom	0.999	1.019	0.966	0.967	0.948
16	Canada	0.815	0.810	0.819	1.005	1.011
17	Ireland	1.107	1.123	1.093	0.987	0.973
18	Spain	0.931	0.945	0.874	0.939	0.925
19	Greece	1.096	1.111	1.026	0.936	0.923
20	Portugal	1.419	1.447	1.273	0.897	0.880
21	Korea	3.144	3.273	2.381	0.757	0.727
22	Argentina	1.266	1.292	1.092	0.863	0.845
23	Uruguay	0.785	0.801	0.675	0.860	0.843
24	Chile	1.000	1.031	0.943	0.943	0.915
25	Brazil	0.788	0.812	0.674	0.855	0.830
26	Hungary	0.852	0.858	0.733	0.860	0.854
27	Dominica	0.881	0.886	0.868	0.985	0.980
28	Poland	0.684	0.679	0.592	0.865	0.872
29	Panama	0.904	0.932	0.831	0.919	0.892
30	Venezuela	0.460	0.482	0.425	0.924	0.882
31	Botswana	2.073	2.126	1.787	0.862	0.841
32	Peru	0.789	0.828	0.682	0.864	0.824
33	Tunisia	1.055	1.111	1.034	0.980	0.931
34	Ecuador	0.590	0.622	0.474	0.803	0.762
35	Philippines	0.837	0.883	0.768	0.918	0.870
36	Indonesia	1.495	1.636	1.441	0.964	0.881
37	Bolivia	0.651	0.668	0.646	0.992	0.967
38	Nigeria	0.231	0.244	0.300	1.299	1.230
39	Zimbabwe	1.173	1.252	1.535	1.309	1.226
40	Cote d`Ivoire	0.459	0.479	0.552	1.203	1.152
41	Cameroon	0.643	0.689	0.724	1.126	1.051
42	Senegal	0.606	0.620	0.621	1.025	1.002
43	Zambia	0.572	0.573	0.610	1.066	1.065

Table 4 (CONT.): GROWTH IN REAL INCOME AT CONSTANT REFERENCE PRICE VERSUS AT NATIONAL PRICES

	Country	Growth in Expenditure		National Real Growth in Expenditure	Ratio	
		AIDS	QUAIDS		AIDS	QUAIDS
44	Kenya	0.595	0.607	0.657	1.104	1.082
45	Mali	0.873	0.914	0.908	1.040	0.993
46	Madagascar	0.515	0.544	0.414	0.804	0.761
47	Malawi	0.590	0.579	0.648	1.098	1.119
48	Tanzania	0.667	0.673	0.502	0.753	0.746

a. from the last two columns of Table 2;
b. from the last column of Table 3.

Figure 1: Comparison of National Price and Constant-Reference-Price Real Growth